TENTH EDITION

BASIC TECHNICAL MATHEMATICS

ALLYN J. WASHINGTON

Basic Technical Mathematics with Calculus

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TENTH EDITION

Basic Technical Mathematics with Calculus

Allyn J. Washington
Dutchess Community College

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Contents

1 Basic Algebraic Operations **1**

1.1 Numbers 2
1.2 Fundamental Operations of Algebra 6
1.3 Calculators and Approximate Numbers 11
1.4 Exponents 16
1.5 Scientific Notation 21
1.6 Roots and Radicals 24
1.7 Addition and Subtraction of Algebraic Expressions 26
1.8 Multiplication of Algebraic Expressions 30
1.9 Division of Algebraic Expressions 32
1.10 Solving Equations 35
1.11 Formulas and Literal Equations 38
1.12 Applied Word Problems 41

Equations **45**
Quick Chapter Review **46**
Review Exercises **46**
Practice Test **48**

2 Geometry **49**

2.1 Lines and Angles 50
2.2 Triangles 53
2.3 Quadrilaterals 60
2.4 Circles 63
2.5 Measurement of Irregular Areas 67
2.6 Solid Geometric Figures 71

Equations **75**
Quick Chapter Review **76**
Review Exercises **76**
Practice Test **79**

3 Functions and Graphs **80**

3.1 Introduction to Functions 81
3.2 More about Functions 84
3.3 Rectangular Coordinates 89
3.4 The Graph of a Function 91
3.5 Graphs on the Graphing Calculator 96
3.6 Graphs of Functions Defined by Tables of Data 101

Quick Chapter Review **105**
Review Exercises **105**
Practice Test **107**

4 The Trigonometric Functions **108**

4.1 Angles 109
4.2 Defining the Trigonometric Functions 112
4.3 Values of the Trigonometric Functions 115
4.4 The Right Triangle 119
4.5 Applications of Right Triangles 124

Equations **129**
Quick Chapter Review **130**
Review Exercises **130**
Practice Test **134**

5 Systems of Linear Equations; Determinants **135**

5.1 Linear Equations 136
5.2 Graphs of Linear Functions 139
5.3 Solving Systems of Two Linear Equations in Two Unknowns Graphically 143
5.4 Solving Systems of Two Linear Equations in Two Unknowns Algebraically 147
5.5 Solving Systems of Two Linear Equations in Two Unknowns by Determinants 154
5.6 Solving Systems of Three Linear Equations in Three Unknowns Algebraically 159
5.7 Solving Systems of Three Linear Equations in Three Unknowns by Determinants 164

Equations **169**
Quick chapter Review **170**
Review Exercises **171**
Practice Test **174**

6 Factoring and Fractions 175

6.1 Special Products 176
6.2 Factoring: Common Factor and Difference of Squares 179
6.3 Factoring Trinomials 184
6.4 The Sum and Difference of Cubes 190
6.5 Equivalent Fractions 191
6.6 Multiplication and Division of Fractions 196
6.7 Addition and Subtraction of Fractions 200
6.8 Equations Involving Fractions 206
Equations **211**
Quick Chapter Review **211**
Review Exercises **211**
Practice Test **213**

7 Quadratic Equations 214

7.1 Quadratic Equations; Solution by Factoring 215
7.2 Completing the Square 220
7.3 The Quadratic Formula 222
7.4 The Graph of the Quadratic Function 226
Equations **231**
Quick Chapter Review **231**
Review Exercises **231**
Practice Test **233**

8 Trigonometric Functions of Any Angle 234

8.1 Signs of the Trigonometric Functions 235
8.2 Trigonometric Functions of Any Angle 237
8.3 Radians 243
8.4 Applications of Radian Measure 247
Equations **253**
Quick Chapter Review **254**
Review Exercises **254**
Practice Test **256**

9 Vectors and Oblique Triangles 257

9.1 Introduction to Vectors 258
9.2 Components of Vectors 262
9.3 Vector Addition by Components 266
9.4 Applications of Vectors 271
9.5 Oblique Triangles, the Law of Sines 276
9.6 The Law of Cosines 283
Equations **287**
Quick Chapter Review **288**
Review Exercises **288**
Practice Test **290**

10 Graphs of the Trigonometric Functions 291

10.1 Graphs of $y = a \sin x$ and $y = a \cos x$ 292
10.2 Graphs of $y = a \sin bx$ and $y = a \cos bx$ 295
10.3 Graphs of $y = a \sin(bx + c)$ and $y = a \cos(bx + c)$ 298
10.4 Graphs of $y = \tan x$, $y = \cot x$, $y = \sec x$, $y = \csc x$ 302
10.5 Applications of the Trigonometric Graphs 304
10.6 Composite Trigonometric Curves 307
Equations **312**
Quick Chapter Review **312**
Review Exercises **312**
Practice Test **314**

11 Exponents and Radicals 315

11.1 Simplifying Expressions with Integral Exponents 316
11.2 Fractional Exponents 320
11.3 Simplest Radical Form 324
11.4 Addition and Subtraction of Radicals 328
11.5 Multiplication and Division of Radicals 330
Equations **334**
Quick Chapter Review **334**
Review Exercises **334**
Practice Test **336**

12 Complex Numbers 337

12.1 Basic Definitions 338
12.2 Basic Operations with Complex Numbers 341
12.3 Graphical Representation of Complex Numbers 344
12.4 Polar Form of a Complex Number 346
12.5 Exponential Form of a Complex Number 348
12.6 Products, Quotients, Powers, and Roots of Complex Numbers 351
12.7 An Application to Alternating-current (ac) Circuits 357
Equations **363**
Quick Chapter Review **364**

Review Exercises 364
Practice Test 365

13 Exponential and Logarithmic Functions 366

13.1 Exponential Functions 367
13.2 Logarithmic Functions 369
13.3 Properties of Logarithms 373
13.4 Logarithms to the Base 10 378
13.5 Natural Logarithms 381
13.6 Exponential and Logarithmic Equations 384
13.7 Graphs on Logarithmic and Semilogarithmic Paper 388
Equations 392
Quick Chapter Review 392
Review Exercises 392
Practice Test 394

14 Additional Types of Equations and Systems of Equations 395

14.1 Graphical Solution of Systems of Equations 396
14.2 Algebraic Solution of Systems of Equations 399
14.3 Equations in Quadratic Form 403
14.4 Equations with Radicals 406
Quick Chapter Review 410
Review Exercises 410
Practice Test 411

15 Equations of Higher Degree 412

15.1 The Remainder and Factor Theorems; Synthetic Division 413
15.2 The Roots of an Equation 418
15.3 Rational and Irrational Roots 423
Equations 429
Quick Chapter Review 429
Review Exercises 429
Practice Test 430

16 Matrices; Systems of Linear Equations 431

16.1 Matrices: Definitions and Basic Operations 432
16.2 Multiplication of Matrices 436
16.3 Finding the Inverse of a Matrix 441
16.4 Matrices and Linear Equations 446
16.5 Gaussian Elimination 450
16.6 Higher-order Determinants 454
Equations 460
Quick Chapter Review 460
Review Exercises 461
Practice Test 463

17 Inequalities 464

17.1 Properties of Inequalities 465
17.2 Solving Linear Inequalities 469
17.3 Solving Nonlinear Inequalities 474
17.4 Inequalities Involving Absolute Values 481
17.5 Graphical Solution of Inequalities with Two Variables 484
17.6 Linear Programming 487
Equations 491
Quick Chapter Review 491
Review Exercises 491
Practice Test 493

18 Variation 494

18.1 Ratio and Proportion 495
18.2 Variation 499
Equations 505
Quick Chapter Review 505
Review Exercises 505
Practice Test 508

19 Sequences and the Binomial Theorem 509

19.1 Arithmetic Sequences 510
19.2 Geometric Sequences 515
19.3 Infinite Geometric Series 519
19.4 The Binomial Theorem 522
Equations 527
Quick Chapter Review 527
Review Exercises 528
Practice Test 530

20 Additional Topics in Trigonometry 531

20.1 Fundamental Trigonometric Identities 532
20.2 The Sum and Difference Formulas 538
20.3 Double-Angle Formulas 543
20.4 Half-Angle Formulas 547
20.5 Solving Trigonometric Equations 550
20.6 The Inverse Trigonometric Functions 554
Equations 560
Quick Chapter Review 561

Review Exercises **561**
Practice Test **563**

21 Plane Analytic Geometry 564

21.1 Basic Definitions 565
21.2 The Straight Line 569
21.3 The Circle 575
21.4 The Parabola 580
21.5 The Ellipse 584
21.6 The Hyperbola 589
21.7 Translation of Axes 595
21.8 The Second-degree Equation 598
21.9 Rotation of Axes 601
21.10 Polar Coordinates 605
21.11 Curves in Polar Coordinates 609
Equations **612**
Quick Chapter Review **613**
Review Exercises **614**
Practice Test **617**

22 Introduction to Statistics 618

22.1 Frequency Distributions 619
22.2 Measures of Central Tendency 623
22.3 Standard Deviation 627
22.4 Normal Distributions 631
22.5 Statistical Process Control 637
22.6 Linear Regression 642
22.7 Nonlinear Regression 647
Equations **650**
Quick Chapter Review **651**
Review Exercises **651**
Practice Test **654**

23 The Derivative 655

23.1 Limits 656
23.2 The Slope of a Tangent to a Curve 664
23.3 The Derivative 667
23.4 The Derivative as an Instantaneous Rate of Change 671
23.5 Derivatives of Polynomials 675
23.6 Derivatives of Products and Quotients of Functions 680
23.7 The Derivative of a Power of a Function 684
23.8 Differentiation of Implicit Functions 690
23.9 Higher Derivatives 693
Equations **696**
Quick Chapter Review **697**
Review Exercises **697**
Practice Test **699**

24 Applications of the Derivative 700

24.1 Tangents and Normals 701
24.2 Newton's Method for Solving Equations 703
24.3 Curvilinear Motion 707
24.4 Related Rates 711
24.5 Using Derivatives in Curve Sketching 715
24.6 More on Curve Sketching 721
24.7 Applied Maximum and Minimum Problems 726
24.8 Differentials and Linear Approximations 733
Equations **737**
Quick Chapter Review **737**
Review Exercises **737**
Practice Test **741**

25 Integration 742

25.1 Antiderivatives 743
25.2 The Indefinite Integral 745
25.3 The Area Under a Curve 750
25.4 The Definite Integral 755
25.5 Numerical Integration: The Trapezoidal Rule 758
25.6 Simpson's Rule 761
Equations **765**
Quick Chapter Review **765**
Review Exercises **765**
Practice Test **767**

26 Applications of Integration 768

26.1 Applications of the Indefinite Integral 769
26.2 Areas by Integration 773
26.3 Volumes by Integration 779
26.4 Centroids 784
26.5 Moments of Inertia 790
26.6 Other Applications 795
Equations **800**
Quick Chapter Review **802**
Review Exercises **802**
Practice Test **804**

27 Differentiation of Transcendental Functions 805

27.1 Derivatives of the Sine and Cosine Functions 806
27.2 Derivatives of the Other Trigonometric Functions 810
27.3 Derivatives of the Inverse Trigonometric Functions 813
27.4 Applications 816
27.5 Derivative of the Logarithmic Function 821
27.6 Derivative of the Exponential Function 825

27.7 L'Hospital's Rule 828
27.8 Applications 832
Equations **835**
Quick Chapter Review **836**
Review Exercises **836**
Practice Test **839**

28 Methods of Integration 840

28.1 The General Power Formula 841
28.2 The Basic Logarithmic Form 843
28.3 The Exponential Form 847
28.4 Basic Trigonometric Forms 850
28.5 Other Trigonometric Forms 854
28.6 Inverse Trigonometric Forms 858
28.7 Integration by Parts 862
28.8 Integration by Trigonometric Substitution 866
28.9 Integration by Partial Fractions: Nonrepeated Linear Factors 869
28.10 Integration by Partial Fractions: Other Cases 872
28.11 Integration by Use of Tables 877
Equations **880**
Quick Chapter Review **881**
Review Exercises **881**
Practice Test **883**

29 Partial Derivatives and Double Integrals 884

29.1 Functions of Two Variables 885
29.2 Curves and Surfaces in Three Dimensions 888
29.3 Partial Derivatives 894
29.4 Double Integrals 898
Equations **902**
Quick Chapter Review **902**
Review Exercises **902**
Practice Test **903**

30 Expansion of Functions in Series 904

30.1 Infinite Series 905
30.2 Maclaurin Series 909
30.3 Operations with Series 913
30.4 Computations by Use of Series Expansions 917
30.5 Taylor Series 920
30.6 Introduction to Fourier Series 923
30.7 More About Fourier Series 930
Equations **935**
Quick Chapter Review **936**
Review Exercises **937**
Practice Test **938**

31 Differential Equations 939

31.1 Solutions of Differential Equations 940
31.2 Separation of Variables 942
31.3 Integrating Combinations 945
31.4 The Linear Differential Equation of the First Order 947
31.5 Numerical Solutions of First-order Equations 950
31.6 Elementary Applications 953
31.7 Higher-order Homogeneous Equations 959
31.8 Auxiliary Equation with Repeated or Complex Roots 963
31.9 Solutions of Nonhomogeneous Equations 966
31.10 Applications of Higher-order Equations 971
31.11 Laplace Transforms 978
31.12 Solving Differential Equations by Laplace Transforms 983
Equations **987**
Quick Chapter Review **988**
Review Exercises **988**
Practice Test **990**

APPENDIX A Solving Word Problems A.1

APPENDIX B Units of Measurement; The Metric System A.2

B.1 Introduction A.2
B.2 Reductions and Conversions A.5

APPENDIX C The Graphing Calculator A.8

C.1 Introduction A.8
C.2 The Graphing Calculator A.8
C.3 Graphing Calculator Programs A.12
C.4 The Advanced Graphing Calculator A.16

APPENDIX D Newton's Method A.24

APPENDIX E A Table of Integrals A.25

Answers to Odd-Numbered Exercises and Quick Chapter Reviews B.1

Solutions to Practice Test Problems C.1

Index of Applications D.1

Index of Writing Exercises D.11

Index D.13

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Preface

Scope of the Book

Basic Technical Mathematics with Calculus, Tenth Edition, is intended primarily for students in technical and pre-engineering technical programs or other programs for which coverage of basic mathematics is required.

Chapters 1 through 20 provide the necessary background for further study with an integrated treatment of algebra and trigonometry. Chapter 21 covers the basic topics of analytic geometry, and Chapter 22 gives an introduction to statistics. Fundamental topics of calculus are covered in Chapters 23 through 31. In the examples and exercises, numerous applications from the various fields of technology are included, primarily to indicate where and how mathematical techniques are used. However, it is not necessary that the student have a specific knowledge of the technical area from which any given problem is taken.

Most students using this text will have a background that includes some algebra and geometry. However, the material is presented in adequate detail for those who may need more study in these areas. The material presented here is sufficient for three to four semesters.

One of the principal reasons for the arrangement of topics in this text is to present material in an order that allows a student to take courses concurrently in allied technical areas, such as physics and electricity. These allied courses normally require a student to know certain mathematics topics by certain definite times; yet the traditional order of topics in mathematics courses makes it difficult to attain this coverage without loss of continuity. However, the material in this book can be rearranged to fit any appropriate sequence of topics. Another feature of this text is that certain topics traditionally included for mathematical completeness have been covered only briefly or have been omitted.

The approach used in this text is not unduly rigorous mathematically, although all appropriate terms and concepts are introduced as needed and given an intuitive or algebraic foundation. The aim is to help the student develop an understanding of mathematical methods without simply providing a collection of formulas. The text material is developed recognizing that it is essential for the student to have a sound background in algebra and trigonometry in order to understand and succeed in any subsequent work in mathematics.

New to This Edition

The tenth edition of *Basic Technical Mathematics with Calculus* includes all the basic features of the earlier editions. Specifically, among the new features of this edition are the following:

- Many sections include revised explanatory material.
- Many examples have been rewritten.
- More examples now include technical applications.
- New exercises are included in nearly all sections.
- A new feature called *Quick Chapter Review* has been added.

- Changing units is now briefly introduced in an example in Section 1.4 with a few margin examples in Chapters 1 and 2. The more complete coverage is included in Appendix B as in earlier editions.

NEW AND REVISED COVERAGE

A *Quick Chapter Review* before the Review Exercises of each chapter includes several brief true or false questions. Each question actively involves the student in the review and can be answered quickly when the student recognizes the topic covered.

THE GRAPHING CALCULATOR

The graphing calculator is used throughout the text. The advanced graphing calculator (TI-89) is included primarily for use in the calculus chapters, but also now included are margin references to its use in the earlier chapters. There are over 270 calculator screens.

The coverage starts in Section 1.3, where it is used for calculational purposes, and its use for graphing starts in Section 3.5. Additional coverage of calculators, including 22 graphing calculator programs, is found in Appendix C.

NEW EXERCISES AND EXAMPLES

There are over 2300 new exercises, including over 500 that illustrate technical and scientific applications. There is a total of over 13,800 exercises, including over 3000 applied exercises, in the tenth edition.

There is a total of over 1400 worked examples, including over 350 that illustrate technical and scientific applications. Of the applied examples, over 60 are new to the tenth edition.

Continuing Features

PAGE LAYOUT

Special attention has been given to the page layout. Nearly all examples are started and completed on the same page (of the 1400 examples, there are 9 exceptions, all but one of which are presented on facing pages). Also, all figures are shown immediately adjacent to the material in which they are discussed.

CHAPTER INTRODUCTIONS

Each chapter introduction illustrates specific examples of how the development of technology has been related to the development of mathematics. In these introductions, it is shown that these past discoveries in technology led to some of the methods in mathematics, whereas in other cases mathematical topics already known were later very useful in bringing about advances in technology.

SPECIAL EXPLANATORY COMMENTS

Throughout the book, special explanatory comments in color have been used in the examples to emphasize and clarify certain important points. Arrows are often used to indicate clearly the part of the example to which reference is made.

PROBLEM SOLVING TECHNIQUES

Techniques and procedures that summarize the approaches in solving many types of problems have been clearly outlined in color-shaded boxes.

IMPORTANT FORMULAS

Throughout the book, important formulas are set off and displayed so that they can be easily referenced for use.

SUBHEADS AND KEY TERMS

Many sections include subheads to indicate where the discussion of a new topic starts within the section. Other key terms are noted in the margin for emphasis and easy reference.

SPECIAL CAUTION AND NOTE INDICATORS

CAUTION
NOTE

Two special margin indicators (as shown at the left) are used. The caution indicator identifies errors students commonly make or places where they frequently have difficulty. The note indicator points out material that is of particular importance in developing or understanding the topic under discussion. There are now over 450 of these indicators, an increase of over 150.

CHAPTER AND SECTION CONTENTS

A listing of section titles for each chapter is given on the introductory page of the chapter. Also, a listing of the key topics of each section is given below the section number and title on the first page of the section. This gives the student and instructor a quick preview of the chapter and section contents.

EXAMPLE DESCRIPTIONS

A brief descriptive title is given with each example number. This gives an easy reference for the example, particularly when reviewing the contents of the section.

PRACTICE EXERCISES

Most sections include some *practice exercises* in the margin. They are included so that a student is more actively involved in the learning process and can check his or her understanding of the material. They can also be used for classroom exercises. The answers to these exercises are given at the end of the exercises set for the section. There are over 450 of these exercises, of which over 100 are new to the tenth edition.

EXERCISES DIRECTLY REFERENCED TO TEXT EXAMPLES

The first few exercises in most of the text sections are referenced directly to a specific example of the section. These examples are worded so that it is necessary for the student to refer to the example in order to complete the required solution. In this way, the student should be able to better review and understand the text material before attempting to solve the exercises that follow.

WRITING EXERCISES

One specific writing exercise is included at the end of each chapter. These exercises give the student practice in explaining their solutions. Also there are over 420 additional exercises through the book (at least seven in each chapter) that require at least a sentence or two of explanation as part of the answer. These are noted by the W symbol next to the exercise number. A special index of Writing Exercises is included at the back of the book.

WORD PROBLEMS

There are over 130 examples throughout the text that show complete solutions of word problems. Of these over 20 are new to the tenth edition. There are nearly 1000 exercises, of which over 200 are new, in which word problems are to be solved.

EQUATIONS, CHAPTER REVIEW, REVIEW EXERCISES, PRACTICE TESTS

At the end of each chapter, all important equations are listed together for easy reference. Each chapter is also followed by a Quick Chapter Review (as previously noted) and a set of review exercises that covers all the material in the chapter. Following the review exercises is a chapter practice test that students can use to check their understanding of the material. Solutions to all practice test problems are given in the back of the book.

APPLICATIONS AND UNITS OF MEASUREMENTS

Examples and exercises illustrate the application of mathematics in all fields of technology. Many relate to modern technology such as computer design, electronics, solar energy, lasers fiber optics, the environment, and space technology. Others examples and exercises relate to technologies such as aeronautics, architecture, automotive, business, chemical, civil, construction, energy, environmental, fire science, machine, medical, meteorology, navigation, police, refrigeration, seismology, and wastewater.

FIGURES

There are over 1600 figures in the text, over 110 (including over 40 new calculator screens) of which are new to the tenth edition.

MARGIN NOTES

Throughout the text, some margin notes point out relevant historical events in mathematics and technology. Other margin notes are used to make specific comments related to the text material. Also, where appropriate, equations from earlier material are shown for reference in the margin. There is a total of over 430 of these notes, of which over 60 are new to the tenth edition.

ANSWERS TO EXERCISES

The answers to all odd-numbered exercises (except the end-of-chapter writing exercises) are given near the end of the book. The *Student's Solution Manual* contains solutions to every other odd-numbered exercise and the *Instructor's Solution Manual* contains solutions to all section exercises.

FLEXIBILITY OF MATERIAL COVERAGE

The order of coverage can be changed in many places and certain sections may be omitted without loss of continuity of coverage. Users of earlier editions have indicated successful use of numerous variations in coverage. Any changes will depend on the type of course and completeness required.

Supplements

SUPPLEMENTS FOR THE INSTRUCTOR

To access supplementary materials online, instructors need to request an instructor access code. Go to **www.pearsonhighered.com/irc**, where you can register for an instructor access code. Within 48 hours after registering, you will receive a confirming email including an instructor access code. Once you have received your code, go to the site and log on for full instructions on downloading the materials you wish to use.

Instructor's Solutions Manual
The *Instructor's Solution Manual* by Bob Martin contains detailed solutions to every section exercise, including review exercises.

TestGen with Algorithmically Generated Questions
Instructors can easily create tests from textbook section objectives. Algorithmically generated questions allow unlimited versions. Instructors can edit problems or create their own using the built-in question editor to generate graphs, import graphics, and insert math notation, insert variable numbers, or text. Tests can be printed or administered online via the Web or other network.

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SUPPLEMENT FOR THE STUDENT

Student's Solutions Manual (ISBN: 0-13-325351-1)
The *Student's Solutions Manual* by Bob Martin includes detailed solutions for every other odd-numbered section exercise.

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Finally, I wish to sincerely thank again each of the over 350 reviewers of the ten editions of this text. Their comments have helped further the education of more than two million students during the fifty years of this text since it was first published in 1964.

A.J.W.

Basic Technical Mathematics with Calculus

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[1] Basic Algebraic Operations

1.1 Numbers
1.2 Fundamental Operations of Algebra
1.3 Calculators and Approximate Numbers
1.4 Exponents
1.5 Scientific Notation
1.6 Roots and Radicals
1.7 Addition and Subtraction of Algebraic Expressions
1.8 Multiplication of Algebraic Expressions
1.9 Division of Algebraic Expressions
1.10 Solving Equations
1.11 Formulas and Literal Equations
1.12 Applied Word Problems

CHAPTER EQUATIONS
QUICK CHAPTER REVIEW
REVIEW EXERCISES
PRACTICE TEST

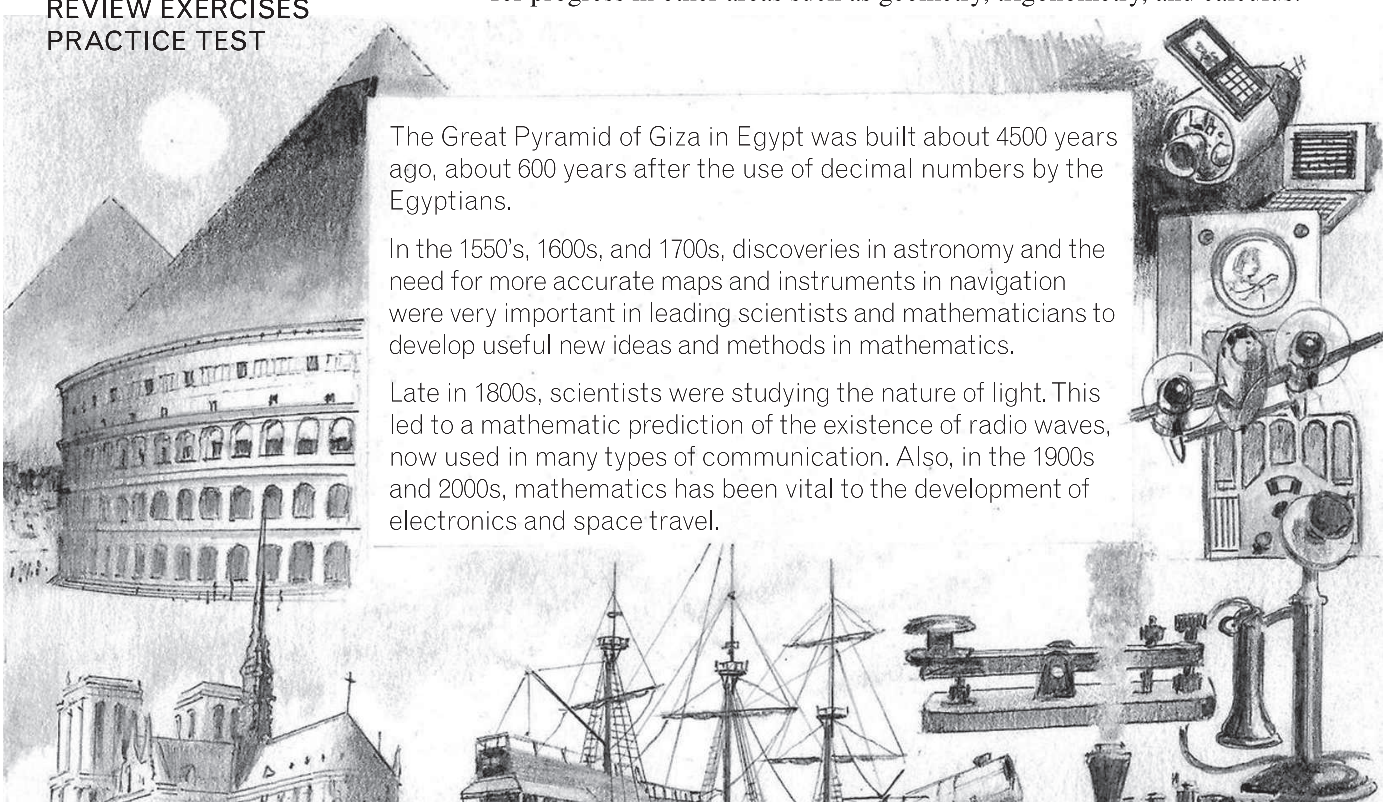
Interest in things such as the land on which they lived, the structures they built, and the motion of the planets led people in early civilizations to keep records and to create methods of counting and measuring. In turn, some of the early ideas of arithmetic, geometry, and trigonometry were developed. From such beginnings, mathematics has played a key role in the great advances in science and technology.

Often, mathematical methods were developed from studies made in sciences, such as astronomy and physics, to better describe and understand the subject being studied. Some of these methods resulted from the needs in a particular area of application.

Many people were interested in the math itself and added to what was then known. Although this additional mathematical knowledge may not have been related to applications at the time it was developed, it often later became useful in applied areas.

In the chapter introductions that follow, examples of the interaction of technology and mathematics are given. From these examples and the text material, it is hoped you will better understand the important role that math has had and still has in technology. In this text, there are applications from technologies including (but not limited to) aeronautical, business, communications, electricity, electronics, engineering, environmental, heat and air conditioning, mechanical, medical, meteorology, petroleum, product design, solar, and space. To solve the applied problems in this text will require a knowledge of the mathematics presented but will *not* require prior knowledge of the field of application.

We begin by reviewing the concepts that deal with numbers and symbols. This will enable us to develop topics in algebra, an understanding of which is essential for progress in other areas such as geometry, trigonometry, and calculus.



The Great Pyramid of Giza in Egypt was built about 4500 years ago, about 600 years after the use of decimal numbers by the Egyptians.

In the 1550's, 1600s, and 1700s, discoveries in astronomy and the need for more accurate maps and instruments in navigation were very important in leading scientists and mathematicians to develop useful new ideas and methods in mathematics.

Late in 1800s, scientists were studying the nature of light. This led to a mathematic prediction of the existence of radio waves, now used in many types of communication. Also, in the 1900s and 2000s, mathematics has been vital to the development of electronics and space travel.

1.1 Numbers

Real Number System • Number Line • Absolute Value • Signs of Inequality • Reciprocal • Denominate Numbers • Literal Numbers

In technology and science, as well as in everyday life, we use the very familiar **counting numbers,** or **natural numbers** 1, 2, 3, and so on. Because it is necessary and useful to use negative numbers as well as positive numbers in mathematics and its applications, the natural numbers are called the **positive integers,** and the numbers $-1, -2, -3$, and so on are the **negative integers.**

Therefore, *the* **integers** *include the positive integers, the negative integers*, and **zero,** *which is neither positive nor negative*. This means that the integers are the numbers . . . , $-3, -2, -1, 0, 1, 2, 3$ . . . and so on.

A **rational number** *is a number that can be expressed as the division of one integer* ***a*** *by another nonzero integer* ***b****, and can be represented by the fraction* $\boldsymbol{a/b}$. Here ***a*** *is the* **numerator** *and* ***b*** *is the* **denominator.** Here we have used algebra by letting letters represent numbers.

■ Irrational numbers were discussed by the Greek mathematician Pythagoras in about 540 B.C.E.

Another type of number, an **irrational number,** *cannot be written in the form of a fraction that is the division of one integer by another integer*. The following example illustrates integers, rational numbers, and irrational numbers.

EXAMPLE 1 Identifying rational numbers and irrational numbers

The numbers 5 and -19 are integers. They are also rational numbers because they can be written as $\frac{5}{1}$ and $\frac{-19}{1}$, respectively. Normally, we do not write the 1's in the denominators.

The numbers $\frac{5}{8}$ and $\frac{-11}{3}$ are rational numbers because the numerator and the denominator of each are integers.

■ For reference, $\pi = 3.14159265\ldots$

The numbers $\sqrt{2}$ and π are irrational numbers. It is not possible to find two integers, one divided by the other, to represent either of these numbers. It can be shown that square roots (and other roots) that cannot be expressed exactly in decimal form are irrational. Also, $\frac{22}{7}$ is sometimes used as an *approximation* for π, but it is not equal *exactly* to π. We must remember that $\frac{22}{7}$ is rational and π is irrational.

■ A notation that is often used for repeating decimals is to place a bar over the digits that repeat. Using this notation we can write $\frac{1121}{1665} = 0.6\overline{732}$ and $\frac{2}{3} = 0.\overline{6}$.

The decimal number 1.5 is rational since it can be written as $\frac{3}{2}$. Any such *terminating decimal* is rational. The number $0.6666\ldots$, where the 6's continue on indefinitely, is rational because we may write it as $\frac{2}{3}$. In fact, any *repeating decimal* (in decimal form, a specific sequence of digits is repeated indefinitely) is rational. The decimal number $0.6732732732\ldots$ is a repeating decimal where the sequence of digits 732 is repeated indefinitely $(0.6732732732\ldots = \frac{1121}{1665})$. ■

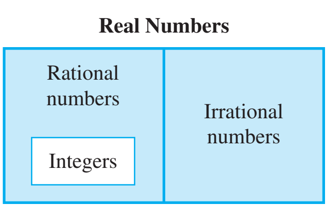


Fig. 1.1

The integers, the rational numbers, and the irrational numbers, including all such numbers that are positive, negative, or zero, make up the **real number system** (see Fig. 1.1). There are times we will encounter an **imaginary number,** *the name given to the square root of a negative number*. Imaginary numbers are not real numbers and will be discussed in Chapter 12. However, unless specifically noted, we will use real numbers. Until Chapter 12, it will be necessary to only *recognize* imaginary numbers when they occur.

Also in Chapter 12, we will consider **complex numbers,** which include both the real numbers and imaginary numbers. See Exercise 37 of this section.

■ Real numbers and imaginary numbers are both included in the *complex number system*. See Exercise 37.

EXAMPLE 2 Identifying real numbers and imaginary numbers

The number 7 is an integer. It is also rational because $7 = \frac{7}{1}$, and it is a real number since the real numbers include all the rational numbers.

The number 3π is irrational, and it is real because the real numbers include all the irrational numbers.

The numbers $\sqrt{-10}$ and $-\sqrt{-7}$ are imaginary numbers.

The number $\frac{-3}{7}$ is rational and real. The number $-\sqrt{7}$ is irrational and real.

The number $\frac{\pi}{6}$ is irrational and real. The number $\frac{\sqrt{-3}}{2}$ is imaginary. ■

■ Fractions were used by early Egyptians and Babylonians. They were used for calculations that involved parts of measurements, property, and possessions.

A **fraction** *may contain any number or symbol representing a number in its numerator or in its denominator*. The fraction indicates the division of the numerator by the denominator, as we previously indicated in writing rational numbers. Therefore, a fraction may be a number that is rational, irrational, or imaginary.

EXAMPLE 3 Fractions

The numbers $\frac{2}{7}$ and $\frac{-3}{2}$ are fractions, and they are rational.

The numbers $\frac{\sqrt{2}}{9}$ and $\frac{6}{\pi}$ are fractions, but they are not rational numbers. It is not possible to express either as one integer divided by another integer.

The number $\frac{\sqrt{-5}}{6}$ is a fraction, and it is an imaginary number. ■

The Number Line

Real numbers may be represented by points on a line. We draw a horizontal line and designate some point on it by O, which we call the **origin** (see Fig. 1.2). The integer *zero* is located at this point. Equal intervals are marked to the right of the origin, and the positive integers are placed at these positions. The other positive rational numbers are located between the integers. The points that cannot be defined as rational numbers represent irrational numbers. We cannot tell whether a given point represents a rational number or an irrational number unless it is specifically marked to indicate its value.

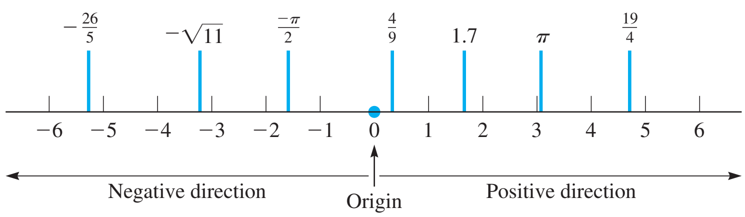


Fig. 1.2

The negative numbers are located on the number line by starting at the origin and marking off equal intervals *to the left, which is the* **negative direction.** As shown in Fig. 1.2, *the positive numbers are to the right of the origin and the negative numbers are to the left of the origin*. Representing numbers in this way is especially useful for graphical methods.

We next define another important concept of a number. *The* **absolute value** *of a positive number is the number itself, and the absolute value of a negative number is the corresponding positive number*. On the number line, we may interpret the absolute value of a number as the distance (which is always positive) between the origin and the number. Absolute value is denoted by writing the number between vertical lines, as shown in the following example.

EXAMPLE 4 Absolute value

The absolute value of 6 is 6, and the absolute value of -7 is 7. We write these as $|6| = 6$ and $|-7| = 7$. See Fig. 1.3.

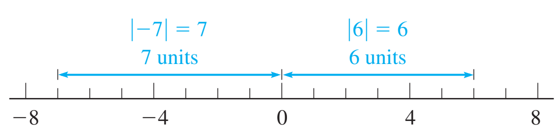


Fig. 1.3

Other examples are $\left|\frac{7}{5}\right| = \frac{7}{5}$, $|-\sqrt{2}| = \sqrt{2}$, $|0| = 0$, $-|\pi| = -\pi$, $|-5.29| = 5.29$, $-|-9| = -9$ since $|-9| = 9$. ■

Practice Exercises

1. $|-4.2| = ?$ **2.** $-\left|-\frac{3}{4}\right| = ?$

■ The symbols $=$, $<$, and $>$ were introduced by English mathematicians in the late 1500s.

On the number line, *if a first number is to the right of a second number, then the first number is said to be* **greater than** *the second. If the first number is to the left of the second, it is* **less than** *the second number*. The symbol $>$ designates "is greater than," and the symbol $<$ designates "is less than." These are called **signs of inequality.** See Fig. 1.4.

EXAMPLE 5 Signs of inequality

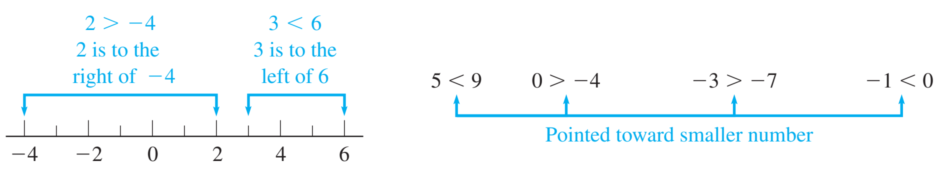


Fig. 1.4 ■

Practice Exercises

Place the correct sign of inequality ($<$ or $>$) between the given numbers.

3. $-5 \quad 4$ **4.** $0 \quad -3$

Every number, except zero, has a **reciprocal.** *The reciprocal of a number is 1 divided by the number.*

EXAMPLE 6 Reciprocal

Practice Exercise

5. Find the reciprocals of

(a) -4 **(b)** $\frac{3}{8}$

The reciprocal of 7 is $\frac{1}{7}$. The reciprocal of $\frac{2}{3}$ is

$$\frac{1}{\frac{2}{3}} = 1 \times \frac{3}{2} = \frac{3}{2} \qquad \text{invert denominator and multiply (from arithmetic)}$$

The reciprocal of 0.5 is $\frac{1}{0.5} = 2$. The reciprocal of $-\pi$ is $-\frac{1}{\pi}$. Note that the negative sign is retained in the reciprocal of a negative number.

We showed the multiplication of 1 and $\frac{3}{2}$ as $1 \times \frac{3}{2}$. We could also show it as $1 \cdot \frac{3}{2}$ or $1\left(\frac{3}{2}\right)$. We will often find the form with parentheses is preferable. ■

In applications, *numbers that represent a measurement and are written with units of measurement are called* **denominate numbers.** The next example illustrates the use of units and the symbols that represent them.

EXAMPLE 7 Denominate numbers

■ For reference, see Appendix B for units of measurement and the symbols used for them.

To show that a certain TV weighs 62 pounds, we write the weight as 62 lb.

To show that a giant redwood tree is 330 feet high, we write the height as 300 ft.

To show that the speed of a rocket is 1500 meters per second, we write the speed as 1500 m/s. (Note the use of s for second. We use s rather than sec.)

To show that the area of a computer chip is 0.75 square inch, we write the area as 0.75 in.^{2}. (We will not use sq in.)

To show that the volume of water in a glass tube is 25 cubic centimeters, we write the volume as 25 cm^3. (We will not use cu cm nor cc.) ■

Literal Numbers

It is usually more convenient to state definitions and operations on numbers in a general form. *To do this, we represent the numbers by letters, called* **literal numbers.** For example, if we want to say "If a first number is to the right of a second number on the number line, then the first number is greater than the second number," we can write "If a is to the right of b on the number line, then $a > b$." Another example of using a literal number is "The reciprocal of n is $1/n$."

Certain literal numbers may take on any allowable value, whereas other literal numbers represent the same value throughout the discussion. *Those literal numbers that may vary in a given problem are called* **variables,** and *those literal numbers that are held fixed are called* **constants.**

EXAMPLE 8 Variables and constants

(a) The resistance of an electric resistor is R. The current I in the resistor equals the voltage V divided by R, written as $I = V/R$. For this resistor, I and V may take on various values, and R is fixed. This means I and V are variables and R is a constant. For a *different* resistor, the value of R may differ.

(b) The fixed cost for a calculator manufacturer to operate a certain plant is b dollars per day, and it costs a dollars to produce each calculator. The total daily cost C to produce n calculators is

$$C = an + b$$

Here, C and n are variables, and a and b are constants, and the product of a and n is shown as *an.* For *another* plant, the values of a and b would probably differ.

If specific numerical values of a and b are known, say $a = \$7$ per calculator and $b = \$3000$, then $C = 7n + 3000$. Thus, constants may be numerical or literal. ■

EXERCISES 1.1

In Exercises 1–4, make the given changes in the indicated examples of this section, and then answer the given questions.

1. In the first line of Example 1, change the 5 to -3 and the -19 to 14. What other changes must then be made in the first paragraph?

2. In Example 4, change the 6 to -6. What other changes must then be made in the first paragraph?

3. In the left figure of Example 5, change the 2 to -6. What other changes must then be made?

4. In Example 6, change the $\frac{2}{3}$ to $\frac{3}{2}$. What other changes must then be made?

In Exercises 5 and 6, designate each of the given numbers as being an integer, rational, irrational, real, or imaginary. (More than one designation may be correct.)

5. $3, \quad \sqrt{-4}, \quad -\dfrac{\pi}{6}, \quad \dfrac{1}{8}$ 　　**6.** $-\sqrt{-6}, \quad -2.33, \quad \dfrac{\sqrt{7}}{3}, \quad -6$

In Exercises 7 and 8, find the absolute value of each real number.

7. $3, \quad -4, \quad -\dfrac{\pi}{2}, \quad \sqrt{-1}$ 　　**8.** $-0.857, \quad \sqrt{2}, \quad -\dfrac{19}{4}, \quad \dfrac{\sqrt{-5}}{-2}$

In Exercises 9–16, insert the correct sign of inequality ($>$ or $<$) between the given numbers.

9. $6 \qquad 8$ 　　**10.** $7 \qquad 5$

11. $-\pi \qquad -3.1416$ 　　**12.** $-4 \qquad 0$

13. $-4 \qquad -|-3|$ 　　**14.** $-\sqrt{2} \qquad -1.42$

15. $-\dfrac{1}{3} \qquad -\dfrac{1}{2}$ 　　**16.** $-0.6 \qquad 0.2$

In Exercises 17 and 18, find the reciprocal of each number.

17. $3, \quad -\dfrac{4}{\sqrt{3}}, \quad \dfrac{y}{b}$ 　　**18.** $-\dfrac{1}{3}, \quad 0.25, \quad 2x$

In Exercises 19 and 20, locate (approximately) each number on a number line as in Fig. 1.2.

19. $2.5, \quad -\dfrac{12}{5}, \quad \sqrt{3}, \quad -\dfrac{3}{4}$ 　　**20.** $-\dfrac{\sqrt{2}}{2}, \quad 2\pi, \quad \dfrac{123}{19}, \quad -\dfrac{7}{3}$

In Exercises 21–44, solve the given problems. Refer to Appendix B for units of measurement and their symbols.

21. Is an absolute value always positive? Explain.

22. Is -2.17 rational? Explain.

23. What is the reciprocal of the reciprocal of any positive or negative number?

24. Find a rational number between -0.9 and -1.0 that can be written with a denominator of 11 and an integer in the numerator.

25. Find a rational number between 0.13 and 0.14 that can be written with a numerator of 3 and an integer in the denominator.

26. If $b > a$ and $a > 0$, is $|b - a| < |b| - |a|$?

27. List the following numbers in numerical order, starting with the smallest: $-1, 9, \pi, \sqrt{5}, |-8|, -|-3|, -3.1$.

28. List the following numbers in numerical order, starting with the smallest: $\frac{1}{5}, -\sqrt{10}, -|-6|, -4, 0.25, |-\pi|$.

29. If a and b are positive integers and $b > a$, what type of number is represented by the following?

(a) $b - a$ 　**(b)** $a - b$ 　**(c)** $\dfrac{b - a}{b + a}$

30. If a and b represent positive integers, what kind of number is represented by (a) $a + b$, (b) a/b, and (c) $a \times b$?

31. For any positive or negative integer: (a) Is its absolute value always an integer? (b) Is its reciprocal always a rational number?

32. For any positive or negative rational number: (a) Is its absolute value always a rational number? (b) Is its reciprocal always a rational number?

33. Describe the location of a number x on the number line when (a) $x > 0$ and (b) $x < -4$.

34. Describe the location of a number x on the number line when (a) $|x| < 1$ and (b) $|x| > 2$.

35. For a number $x > 1$, describe the location on the number line of the reciprocal of x.

36. For a number $x < 0$, describe the location on the number line of the number with a value of $|x|$.

37. A *complex number* is defined as $a + bj$, where a and b are real numbers and $j = \sqrt{-1}$. For what values of a and b is the complex number $a + bj$ a real number? (All real numbers and all imaginary numbers are also complex numbers.)

38. A sensitive gauge measures the total weight w of a container and the water that forms in it as vapor condenses. It is found that $w = c\sqrt{0.1t + 1}$, where c is the weight of the container and t is the time of condensation. Identify the variables and constants.

39. In an electric circuit, the reciprocal of the total capacitance of two capacitors in series is the sum of the reciprocals of the capacitances. Find the total capacitance of two capacitances of 0.0040 F and 0.0010 F connected in series.

40. Alternating-current (ac) voltages change rapidly between positive and negative values. If a voltage of 100 V changes to −200 V, which is greater in absolute value?

41. The memory of a certain computer has a bits in each byte. Express the number N of bits in n kilobytes in an equation. (A *bit* is a single digit, and bits are grouped in *bytes* in order to represent special characters. Generally, there are 8 bits per byte. If necessary, see Appendix B for the meaning of *kilo*.)

42. The computer design of the base of a truss is x ft. long. Later it is redesigned and shortened by y in. Give an equation for the length L, in inches, of the base in the second design.

43. In a laboratory report, a student wrote "−20°C > −30°C." Is this statement correct? Explain.

44. After 5 s, the pressure on a valve is less than 60 lb/in.^{2} (pounds per square inch). Using t to represent time and p to represent pressure, this statement can be written "for $t > 5$ s, $p < 60$ lb/in.^{2}." In this way, write the statement "when the current I in a circuit is less than 4 A, the resistance R is greater than 12 Ω (ohms)."

Answers to Practice Exercises

1. 4.2 **2.** $-\frac{3}{4}$ **3.** $<$ **4.** $>$ **5.** **(a)** $-\frac{1}{4}$ **(b)** $\frac{8}{3}$

1.2 Fundamental Operations of Algebra

Fundamental Laws of Algebra • Operations on Positive and Negative Numbers • Order of Operations • Operations with Zero

The Commutative and Associative Laws

If two numbers are added, it does not matter in which order they are added. (For example, $5 + 3 = 8$ and $3 + 5 = 8$, or $5 + 3 = 3 + 5$.) This statement, generalized and accepted as being correct for all possible combinations of numbers being added, is called the **commutative law** for addition. It states that *the sum of two numbers is the same, regardless of the order in which they are added*. We make no attempt to prove this law in general, but accept that it is true.

In the same way, we have the **associative law** for addition, which states that *the sum of three or more numbers is the same, regardless of the way in which they are grouped for addition*. For example, $3 + (5 + 6) = (3 + 5) + 6$.

The laws just stated for addition are also true for multiplication. Therefore, *the product of two numbers is the same, regardless of the order in which they are multiplied,* and *the product of three or more numbers is the same, regardless of the way in which they are grouped for multiplication*. For example, $2 \times 5 = 5 \times 2$, and $5 \times (4 \times 2) = (5 \times 4) \times 2$.

The Distributive Law

Another very important law is the **distributive law.** It states that *the product of one number and the sum of two or more other numbers is equal to the sum of the products of the first number and each of the other numbers of the sum*. For example,

■ Note carefully the difference:
associative law: $5 \times (4 \times 2)$
distributive law: $5 \times (4 + 2)$

$$5(4 + 2) = 5 \times 4 + 5 \times 2$$

In this case, it can be seen that the total is 30 on each side.

In practice, these **fundamental laws of algebra** are used naturally without thinking about them, except perhaps for the distributive law.

Not all operations are commutative and associative. For example, division is not commutative, because the order of division of two numbers does matter. For instance, $\frac{6}{5} \neq \frac{5}{6}$ ($\neq$ is read "does not equal"). (Also, see Exercise 52.)

Using literal numbers, the fundamental laws of algebra are as follows:

Commutative law of addition: $a + b = b + a$

Associative law of addition: $a + (b + c) = (a + b) + c$

Commutative law of multiplication: $ab = ba$

Associative law of multiplication: $a(bc) = (ab)c$

Distributive law: $a(b + c) = ab + ac$

■ Note the meaning of *identity*.

Each of these laws is an example of an *identity,* in that the expression to the left of the = sign equals the expression to the right for any value of each of a, b, and c.

OPERATIONS ON POSITIVE AND NEGATIVE NUMBERS

When using the basic operations (addition, subtraction, multiplication, division) on positive and negative numbers, we determine the result to be either positive or negative according to the following rules.

Addition of two numbers of the same sign *Add their absolute values and assign the sum their common sign.*

EXAMPLE 1 Adding numbers of the same sign

(a) $2 + 6 = 8$ the sum of two positive numbers is positive

(b) $-2 + (-6) = -(2 + 6) = -8$ the sum of two negative numbers is negative

■ From Section 1.1, we recall that a positive number is preceded by no sign. Therefore, in using these rules, we show the "sign" of a positive number by simply writing the number itself.

The negative number -6 is placed in parentheses because it is also preceded by a plus sign showing addition. It is not necessary to place the -2 in parentheses. ■

Addition of two numbers of different signs *Subtract the number of smaller absolute value from the number of larger absolute value and assign to the result the sign of the number of larger absolute value.*

EXAMPLE 2 Adding numbers of different signs

(a) $2 + (-6) = -(6 - 2) = -4$ the negative 6 has the larger absolute value

(b) $-6 + 2 = -(6 - 2) = -4$

(c) $6 + (-2) = 6 - 2 = 4$ the positive 6 has the larger absolute value

(d) $-2 + 6 = 6 - 2 = 4$

the subtraction of absolute values ■

Subtraction of one number from another *Change the sign of the number being subtracted and change the subtraction to addition. Perform the addition.*

EXAMPLE 3 Subtracting positive and negative numbers

(a) $2 - 6 = 2 + (-6) = -(6 - 2) = -4$

Note that after changing the subtraction to addition, and changing the sign of 6 to make it -6, we have precisely the same illustration as Example 2(a).

(b) $-2 - 6 = -2 + (-6) = -(2 + 6) = -8$

Note that after changing the subtraction to addition, and changing the sign of 6 to make it -6, we have precisely the same illustration as Example 1(b).

(c) $-a - (-a) = -a + a = 0$

Subtraction of a Negative Number

This shows that subtracting a number from itself results in zero, even if the number is negative. Therefore, *subtracting a negative number is equivalent to adding a positive number of the same absolute value*.

(d) $-2 - (-6) = -2 + 6 = +4 = 4$

(e) The change in temperature from $-12°C$ to $-26°C$ is
$-26°C - (-12°C) = -26°C + 12°C = -14°C$ ■

Multiplication and division of two numbers *The product (or quotient) of two numbers of the same sign is positive. The product (or quotient) of two numbers of different signs is negative.*

EXAMPLE 4 Multiplying and dividing positive and negative numbers

Practice Exercises

Evaluate: **1.** $-5 - (-8)$
2. $-5(-8)$

(a) $3(12) = 3 \times 12 = 36$ $\qquad \dfrac{12}{3} = 4$ — result is positive if both numbers are positive

(b) $-3(-12) = 3 \times 12 = 36$ $\qquad \dfrac{-12}{-3} = 4$ — result is positive if both numbers are negative

(c) $3(-12) = -(3 \times 12) = -36$ $\qquad \dfrac{-12}{3} = -\dfrac{12}{3} = -4$ — result is negative if one number is positive and the other is negative

(d) $-3(12) = -(3 \times 12) = -36$ $\qquad \dfrac{12}{-3} = -\dfrac{12}{3} = -4$ ■

ORDER OF OPERATIONS

Often, how we are to combine numbers is clear by grouping the numbers using symbols such as **parentheses,** ( ), the **bar,** ____ , between the numerator and denominator of a fraction, and **vertical lines** for absolute value. Otherwise, for an expression in which there are several operations, we use the following order of operations.

ORDER OF OPERATIONS

1. *Operations within specific groupings are done first.*
2. *Perform multiplications and divisions (from left to right).*
3. *Then perform additions and subtractions (from left to right).*

EXAMPLE 5 Order of operations

■ Note that $20 \div (2 + 3) = \frac{20}{2+3}$, whereas $20 \div 2 + 3 = \frac{20}{2} + 3$.

(a) $20 \div (2 + 3)$ is evaluated by first adding $2 + 3$ and then dividing. The grouping of $2 + 3$ is clearly shown by the parentheses. Therefore, $20 \div (2 + 3) = 20 \div 5 = 4$.

(b) $20 \div 2 + 3$ is evaluated by first dividing 20 by 2 and then adding. No specific grouping is shown, and therefore the division is done before the addition. This means $20 \div 2 + 3 = 10 + 3 = 13$.

CAUTION ➧ **(c)** $16 - 2 \times 3$ is evaluated by ***first multiplying*** 2 ***by*** 3 and then subtracting. We *do* **not** *first subtract* 2 *from* 16. Therefore, $16 - 2 \times 3 = 16 - 6 = 10$.

Practice Exercises

Evaluate: **3.** $12 - 6 \div 2$
4. $16 \div (2 \times 4)$

(d) $16 \div 2 \times 4$ is evaluated by first dividing 16 by 2 and then multiplying. From left to right, the division occurs first. Therefore, $16 \div 2 \times 4 = 8 \times 4 = 32$.

(e) $|3 - 5| - |-3 - 6|$ is evaluated by first performing the subtractions within the absolute value vertical bars, then evaluating the absolute values, and then subtracting. This means that $|3 - 5| - |-3 - 6| = |-2| - |-9| = 2 - 9 = -7$. ■

When evaluating expressions, it is generally more convenient to change the operations and numbers so that the result is found by the addition and subtraction of positive numbers. When this is done, we must remember that

$$a + (-b) = a - b \tag{1.1}$$

$$a - (-b) = a + b \tag{1.2}$$

Practice Exercises

Evaluate: **5.** $2(-3) - \frac{4-8}{2}$

6. $\frac{|5-15|}{2} - \frac{-9}{3}$

EXAMPLE 6 Evaluating numerical expressions

(a) $7 + (-3) - 6 = 7 - 3 - 6 = 4 - 6 = -2$ using Eq. (1.1)

(b) $\frac{18}{-6} + 5 - (-2)(3) = -3 + 5 - (-6) = 2 + 6 = 8$ using Eq. (1.2)

(c) $\frac{|3-15|}{-2} - \frac{8}{4-6} = \frac{12}{-2} - \frac{8}{-2} = -6 - (-4) = -6 + 4 = -2$

(d) $\frac{-12}{2-8} + \frac{5-1}{2(-1)} = \frac{-12}{-6} + \frac{4}{-2} = 2 + (-2) = 2 - 2 = 0$

In illustration (b), we see that the division and multiplication were done before the addition and subtraction. In (c) and (d), we see that the groupings were evaluated first. Then we did the divisions, and finally the subtraction and addition. ■

EXAMPLE 7 Evaluating in an application

A 3000-lb van going at 40 mi/h ran head-on into a 2000-lb car going at 20 mi/h. An insurance investigator determined the velocity of the vehicles immediately after the collision from the following calculation. See Fig. 1.5.

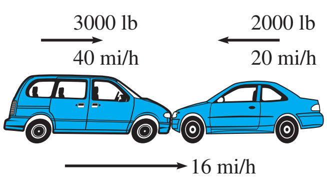


Fig. 1.5

$$\frac{3000(40) + (2000)(-20)}{3000 + 2000} = \frac{120{,}000 + (-40{,}000)}{3000 + 2000} = \frac{120{,}000 - 40{,}000}{5000}$$

$$= \frac{80{,}000}{5000} = 16 \text{ mi/h}$$

The numerator and the denominator must be evaluated before the division is performed. The multiplications in the numerator are performed first, followed by the addition in the denominator and the subtraction in the numerator. ■

OPERATIONS WITH ZERO

Because operations with zero tend to cause some difficulty, we will show them here.

If a is a real number, the operations of addition, subtraction, multiplication, and division with zero are as follows:

$$\mathbf{a + 0 = a}$$

$$\mathbf{a - 0 = a} \qquad \mathbf{0 - a = -a}$$

$$\mathbf{a \times 0 = 0}$$

$$\mathbf{0 \div a = \frac{0}{a} = 0} \qquad (\textbf{if } \mathbf{a \neq 0})$$

($\neq$ means "is not equal to")

EXAMPLE 8 Operations with zero

(a) $5 + 0 = 5$ **(b)** $-6 - 0 = -6$ **(c)** $0 - 4 = -4$

(d) $\frac{0}{6} = 0$ **(e)** $\frac{0}{-3} = 0$ **(f)** $\frac{5 \times 0}{7} = \frac{0}{7} = 0$ ■

Note that there is no result defined for division by zero. To understand the reason for this, consider the results for $\frac{6}{2}$ and $\frac{6}{0}$.

$$\frac{6}{2} = 3 \quad \text{since} \quad 2 \times 3 = 6$$

If $\frac{6}{0} = b$, then $0 \times b = 6$. This cannot be true because $0 \times b = 0$ for any value of b. Thus,

NOTE ▸ ***division by zero is undefined***

(The special case of $\frac{0}{0}$ is termed *indeterminate*. If $\frac{0}{0} = b$, then $0 = 0 \times b$, which is true for any value of b. Therefore, no specific value of b can be determined.)